SAS Honors Program

Capstone Report

Option C

Course 1: Math Finance I, Course 2: Math Finance II

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I. Introduction to Mikhail Fishbeyn and Capstone Decision

When I first arrived at Rutgers University in September 2012, I had no idea what I wanted to do with the next several years of my life. I had been declined or waitlisted from almost every school I applied to, and out of the ones that accepted me (Boston University, Boston College, Carnegie Mellon, etc.) Rutgers was the only affordable option for my family. I am an only child, and a 1st generation American (technically I am an immigrant since I was born in Kiev, Ukraine). My family had high hopes for my future, and even though Rutgers was not my first choice I was still looking forward to starting the next chapter of my life. High school was easy for me, and I had managed to get a scholarship that covered my full tuition. On top of that, I qualified for the School of Arts and Science Honors Program and started my first semester with 17 college credits from AP exams. Despite this, I was very much lost. I had wanted to become an engineer, but due to a mix-up I had never ended up applying for their Honors Program, so I decided to stick with School of Arts and Sciences in the meantime.

I took a variety of courses my first year at Rutgers: Introductory classes in Philosophy, Psychology, Physics, History, and of course Math. My major was listed as undeclared, and I continued to simply take courses to satisfy the CORE requirements while taking higher level math courses. Although each class was unique in its own way, nothing I learned except for Math gave me any sense of purpose. Although I didn't always enjoy learning the material, I enjoyed being challenged and to force myself to become better. My sophomore year followed this same pattern, as I took Honors seminars, writing courses, and continued to learn Math. By this point, I decided to simply finish my Math major since I was well ahead of the credits I needed thanks to taking Calculus I & II in high school. Unfortunately, as I began taking 300 level math courses, I hit a wall in terms of my enjoyment of the material that I could not overcome through my desire to be challenged alone. Discrete Math, Advanced Calculus, Linear Algebra (350), etc. were all math courses that focused heavily on dull, abstract theories and proofs. The things that I had enjoyed about Calculus were not present at all, and many of the classes seemed oriented on creating researchers and math professors. I had begun regretting my decision to major in Mathematics, until I took Math Theory of Probability with Professor Triet Pham.

This class, though called 'theory of probability,' focused much more heavily on real world applications. Things like poker hand probabilities, the Monty Hall problem, normal distributions, etc. all interested me greatly, and it was all thanks to the way Professor Pham taught the course. After the semester ended, I had discussed other courses in the same vein that I could pursue at Rutgers, and Professor Pham recommended the accelerated degree program in Mathematical Finance MS program at Rutgers. After some research, I discovered that this program was exactly what I was looking for: a way to apply both my desire to apply Mathematics in a practical way and a way to make use of my partially-completed Math major. To quote the program's website:

> The accelerated degree program for the Master of Science (MS) in Mathematics with Option in Mathematical Finance is intended for students who wish to earn the MS degree, following their undergraduate degree, with minimum additional time and expense. With careful planning, best begun during the sophomore year, a full-time student can complete an undergraduate degree in four years and the MS in just one academic year (two regular semesters or 9 months).

Aside from a variety of strenuous prerequisite courses (many of which I thankfully completed for my Math major anyway) in Mathematics, Statistics, Economics, and Computer Science, the program also required the completion of 6-12 credits of graduate courses to be taken during my senior year. To complete this program, I planned out a schedule of classes that I would have to take over the course of my junior and senior years. Unfortunately, I quickly realized that I would be cutting it extremely close with the prerequisite courses, along with the various requirements for my Math major, Economics minor, CORE requirements, SAS Honors Program, and job. To fit it all in, I had to make the regrettable choice of not taking Introduction to Math Finance, a course taught by Professor Pham that would prepare me would the graduate courses I will describe later. Although some courses, like Partial Differential Equations and Numerical Analysis I, were indeed a struggle, I managed to succeed in finishing all undergraduate requirements by my senior year. Here, is where the real story of this Capstone begins.

Over the course of my senior year I ended up taking the maximum 12 credits of graduate courses that was allowed by the program. The courses are called Math Finance I, Math Finance II, Numerical Analysis II, and Econometrics I. This means that I would only require an additional 18 credits after graduating to get my Master of Science degree. Since Econometrics I is technically taken through the Rutgers Graduate Economics department, and I haven't taken Numerical Analysis I yet, I decided to choose combined Math Finance I & II as my first and second course for Option C of the Capstone. This has been one of the most challenging years of my life, but I am happy to say that I have completed all of these courses with at least a B and have been officially accepted into the Math Finance Graduate School. In the bulk of this paper, I will reflect on my senior year experiences, discuss what I have learned, and state what I plan to do now that I have completed my Capstone.

II. Brief overview of the material that will be discussed

When I tell people that I am studying Mathematical/Quantitative finance, the first question they usually ask is "what's that?" A brief answer I usually give is that Math Finance is calculus, probability, and advanced statistics applied to various financial markets like bonds, stocks, stock derivatives/options, etc. However, what makes the subject so unique in my eyes is that it is literally the culmination of hundreds of years of mathematics in the modern world. To give some perspective, some of the theorems and formulas used in my textbook, *Stochastic Calculus for Finance II,* were developed as recently as the 1990s. The author of said textbook, Steven E Shreve, put it best when he wrote:

> By awarding Harry Markowitz, William Sharpe and Merton Miller the 1990 Nobel Prize in Economics, the Nobel Prize Committee brought to world-wide attention the fact that the last forty years have seen the emergence of a new scientific discipline, the 'theory of finance.' This theory attempts to understand how financial markets work, how to make them more efficient, and how they should be regulated. It explains and enhances the important role these markets play in reducing risk associated with economic activity. Without losing its application to practical aspects of trading and regulation, the theory of finance has become increasingly mathematical, to the point that problems in finance are now driving research in mathematics (XV).

It gives me a unique feeling to know that in many ways I am officially 'catching up' with the world's known mathematics. It's one thing to learn Calculus, which has been taught for centuries, and another to take a course that only a minute fraction of humanity has ever learned. In the next several sections, I will go into detail about some of the concepts I learned.

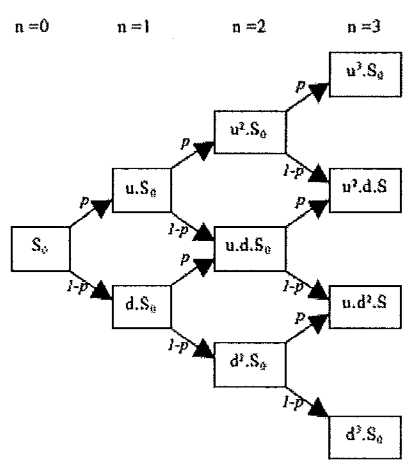
III. Math Finance I

When my senior year began, I was very anxious about the future. By this point I had completed all of my CORE requirements, SAS Honors Program requirements (outside of the Capstone), and only needed one Math class and one Economics class to complete my major and minor. In addition, I had also completed all of the undergraduate prerequisites for the previously stated graduate program and had signed up for my first graduate class: Math Finance I. However, I was still very worried since although I had committed a substantial amount of time towards the Accelerated MSMF program and would be taking courses in the program, I was not officially accepted into the program yet. The application for the program would be due early in the spring, meaning that my performance in Math Finance I would be absolutely crucial since this course, and its continuation Math Finance II, is considered to be the most difficult courses in the entire program. To give an idea about the difficulty of the course, Calculus I-IV, Math Theory of Probability, Partial Differential Equations, Statistics, Finance, and Introduction to Computer Programming are all prerequisites for this course. If I did well in this course, I could expect almost automatic acceptance into the MSMF program since it would show that I am more than capable of completing the entire program. However, the reverse was also true, and I knew that realistically that if I got lower than a B that I could expect to be declined.

Although I had completed all of the prerequisite courses, I still significantly unprepared. As I mentioned previously, I could not take Introduction to Math Finance in my junior year due to scheduling conflicts, and was very intimidated by the scope of the material I would be expected to learn. I consulted with people who have gone through the program, did plenty of outside research, and consulted with the professor of the course, Kihun Nam, on what I should be reviewing.

Finally, the fall 2015 semester arrived and I officially began my senior year. Aside from Math Finance I, I was also taking Advanced Linear Algebra to complete my Math major, Intermediate Microeconomics to complete my Economics minor, and Russia and the West to reach 120 undergraduate credits. Although this only amounted to 12 total credits, I still knew that this was going to be a very strenuous year. The graduate course, Math Finance I, was a 3 hour long lecture from 6:40 to 9:30 PM, and was taught by a Korean professor Kihun Nam. Thankfully, Professor Nam's English was more than satisfactory, and for the first time in a while I felt confident in my ability to succeed in this course.

We began the year by going over the Stock Market and the various derivatives and options that exist because of it. In particular, we focused on options, which are contracts between two or more parties to purchase the right, but not obligation, to buy or sell shares of a stock in the future. Since it's only an option, and not a forward contract, the owner of an option has many different scenarios he must consider to create an optimal result. An important concept we learned early on is the idea of Binomial Asset Pricing, which essentially attempts to model a stock's changes by showing all of its potential paths in discrete (IE: finite) time. The following model describes this phenomenon well:



$$p = \frac{e^{rt/n} - d}{u - d}$$

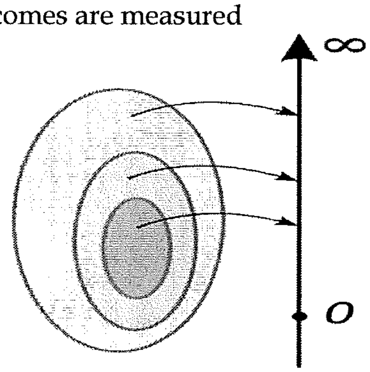
$$u = e^{\sigma\sqrt{t/n}}$$

$$d = e^{-\sigma\sqrt{t/n}}$$

In the above model, a stock is worth some initial quantity S_0, and then either rises in price to u $*S_0$ with probability p, or falls in price to d* S_0 with probability 1-p. Over the course of the model, this same binomial situation repeats itself at n = 1, n= 2, and eventually you are left with the stock's final price. As there are many ways of getting to the same finals price, this is a very useful method for mapping out stocks in finite time. Also, the variables p, u, and d all only depend on σ, which represents the stocks underlying volatility, r, which is the current interest rate, and t, which is time. Given these variables, one could figure out many scenarios for a stock, including when to optimally sell it, what types of options can be made on it and where, etc. However, as the world functions in continuous time, this model isn't particularly useful outside of theory. To create a model that could reflect the continuous nature of the stock market would require me to learn about two main concepts: Measure Theory and Stochastic Calculus.

In this context, Measure Theory refers to all of the various tools I would be using throughout the course. A review of probability theory, random variable analysis, expectation, set theory, and much more was drilled into my head over the course of two three-hour long lectures, and I once again felt completely lost. Complex notations, intricate and abstract concepts, all frustrated me, but I was relieved to learn that we only needed to take away a few concepts for the main topics of the course. The main concepts to take away were about distribution measure, which is a way to assign probability distributions to random events, and filtration, which provides a way to list all possible outcomes of a random variable. Finally, in week four, we began learning Stochastic Calculus.

Visualization of how event outcomes are measured

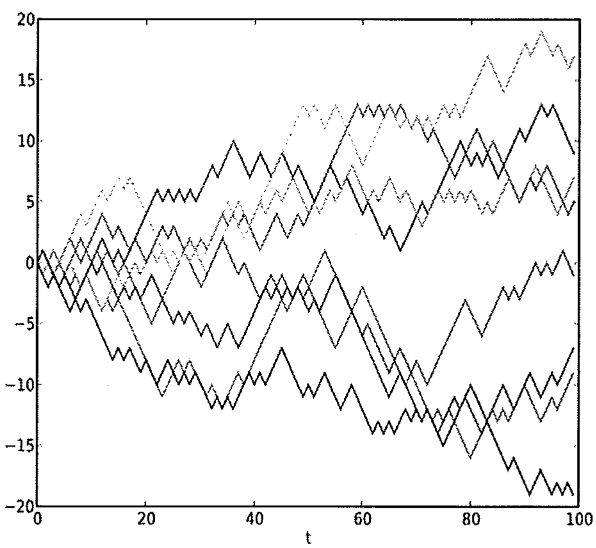


Example of filtration notation

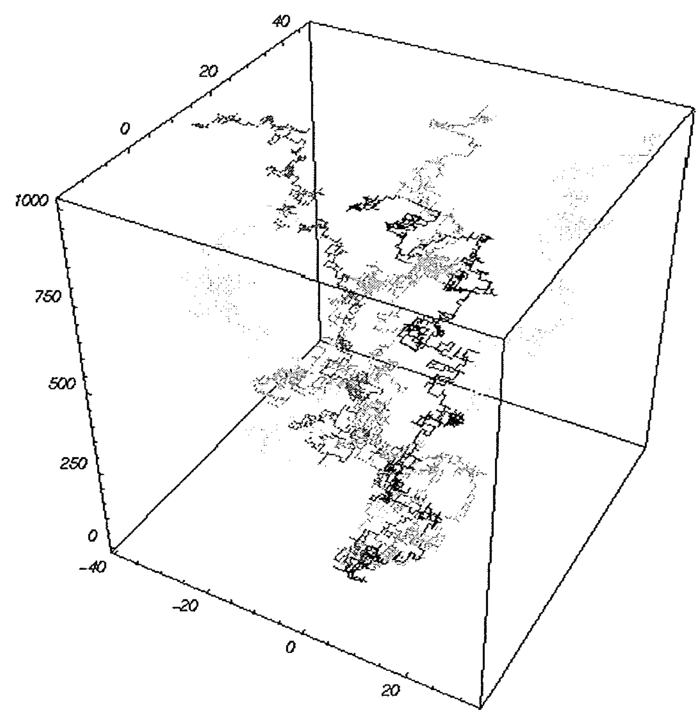
$$F_i^X = \sigma\left\{X_j^{-1}(A)\middle|\, j \in I, j \leq i, A \in \Sigma\right\},$$

To understand what Stochastic Calculus is, you first need to understand the concepts of the ‘random walk’ and Brownian motion. A one dimensional random walk is a simple enough concept to understand: you start at zero, and then either move up or down with equal probability, and is illustrated below. In two dimensions, this becomes a little more complicated, but the concept is still the same. What makes two dimensional random walks interesting is that some paths are clearly followed more than others, as can be seen in the darker, more concentrated areas in the picture below. Accordingly, when evaluating the limit of a two-dimensional random walk, one actually obtains Brownian motion, which was the ultimate goal of looking at random walks in the first place.

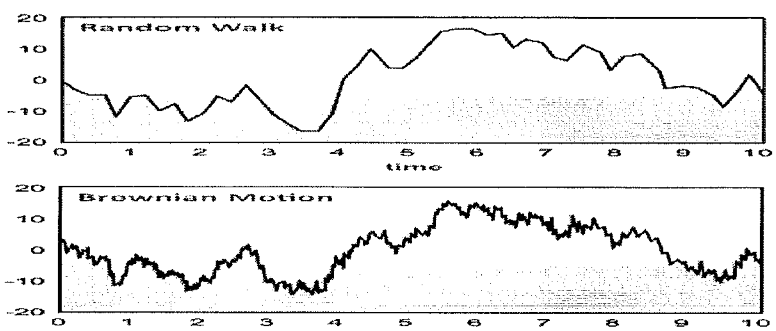
1D Random Walks



2D Random Walks



Brownian motion as limit of random walk



Brownian motion, or more specifically the Weiner Process that characterizes it, is a fundamental concept behind Stochastic Calculus. For a process to be considered Brownian motion it must meet the following requirements:

Definition 3.3. *Let W be a real-valued stochastic process defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The process W is called Brownian motion if it satisfies the following property.*

- *W is continuous with $W_0 = 0$.*
- *For $s \leq t$, the increment $W_t - W_s$ and $\mathcal{F}_s^W$ are independent.*
- *For $s \leq t$, $W_t - W_s \sim \mathcal{N}(0, t - s)$.*

While the first requirement is simple enough, the other two require a bit more explanation. Essentially, the second requirement means that the value of W at a time in the future is not dependent on its time in the past. This is also known as the 'memory-less' property, which makes Brownian motion a Markov Process, which is a process that is only dependent on its previous position and not by its path. A good example of a Markov Process is the position of a Monopoly piece on a game board after each dice roll: your position only depends on where you were before the roll and what you rolled, not the path you took to get there. The third requirement is about distribution, and states that increments of Brownian motion have a normal distribution with a mean of zero and a variance of the differences of the time between the two increments. This, combined with the previous requirement, means that Brownian motion is also a Martingale, which is a process that can be described as a 'fair game' in the sense that information past events cannot be used to predict values of the process. An example of this is a game where you flip a fair coin and win a dollar on heads and lose a dollar on tails. Even if someone flipped a coin 10 times and wrote down each result, it would ultimately not help him at all in determining the result of the next flip. Lastly, Brownian motion has the property of having a quadratic variation that is equal to the time the Brownian motion is being examined it. In other words, this means that Brownian motion is not differentiable with respect to time, which makes sense since otherwise predictions could be made about its near future. Once we had exhausted this topic, Professor introduced us to what is considered the bulk of the course material: Stochastic/Ito Calculus.

To understand Stochastic/Ito Calculus, you must first understand the concept of an Ito Process.

Definition 1.1. *Let W be a Brownian motion. An Itô (diffusion) process is an adapted measurable stochastic process of the form*

$$X_t = X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s$$

where

$$\mathbb{P}\left(\int_0^t |b(\omega, s)| ds < \infty\right) = 1$$
$$\mathbb{P}\left(\int_0^t |\sigma(\omega, s)|^2 ds < \infty\right) = 1$$

for all t.

In short, this definition means that a process X at time t is equal to its original value X_0 and the sums accumulated by the drift term b_s over the interval 0 to t with respect to time and by the volatility term σ_s over the interval 0 to t with respect to the Brownian motion W. Also, both the drift and volatility terms are bounded (IE: finite). This formula for an Ito process X_t is incredibly useful, and is essentially the formula used to describe the behavior of stock prices. What makes this formula unique however is the last integral term, since it is integrated with respect to the path of Brownian motion W. Nothing in my Calculus education taught me how to take a derivative or integral with respect to a random process. Thankfully, there is a formula that simplifies this dilemma.

Theorem 1.3 (Itô-Doeblin formula). *Let X be an Itô process* [1] *and let $f \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}; \mathbb{R})$. Then, for every $t \geq 0$,*

$$f(t, X_t) = f(0, X_0) + \int_0^t \partial_t f(s, X_s) ds + \int_0^t \partial_x f(s, X_s) dX_s + \frac{1}{2} \int_0^t \partial_{xx} f(s, W_s) d[X, X]_s.$$

Moreover, if

$$X_t = X_0 + \int_0^t b_s ds + \int_0^t \sigma_s dW_s,$$

then,

$$f(t, X_t) = f(0, X_0) + \int_0^t \left(\partial_t f(s, X_s) + \partial_x f(s, X_s) b_s + \frac{1}{2} \partial_{xx} f(s, W_s) \sigma_s^2 \right) ds + \int_0^t \partial_x f(s, X_s) \sigma_s dW_s$$

Essentially, the Ito-Doeblin formula is the classic chain rule from calculus applied to a stochastic process. It allows one to write a function of an Ito process (such as the value of a stock) in a way that can be clearly analyzed. A very large portion of my homework and exams was simply rewriting Ito processes using the Ito-Doeblin formula, as it is one of the main skills learned in the course. Likewise, to examine an Ito function's differentiated form, one would use a variation of the Ito formula:

Theorem 1.7. *(Important) Let X be an d-dimensional Itô process, that is b is $\mathbb{R}^d$ valued process, σ is $\mathbb{R}^{d \times n}$ valued process, and W is n-dimensional Brownian motion. Let $f \in C^{1,2}(\mathbb{R}_+ \times \mathbb{R}^d; \mathbb{R})$. Then, for every $t \geq 0$,*

$$\begin{aligned} df(t, X_t) &= \partial_t f(t, X_t) dt + \nabla f(t, X_t) dX_t + \frac{1}{2} (dX_t)^T (Hf)(t, X_t) dX_t \\ &= \partial_t f(t, X_t) dt + \nabla f(t, X_t) dX_t + \frac{1}{2} \sum_{i,j} \partial_{x_i x_j} f(t, X_t) d[X^i, X^j]_t \\ &= \partial_t f(t, X_t) dt + \nabla f(t, X_t) b_t dt + \nabla f(t, X_t) \sigma_t dW_t + \frac{1}{2} \mathbf{tr} \left(\sigma_t^T (Hf)(t, X_t) \sigma_t \right) dt \end{aligned}$$

This is the much more common way of modeling stochastic processes, and is known as a Stochastic Differential Equation (SDE for short). Analysis of SDEs and the various applications of this analysis is the next topic we learned, and it covered most of the semester.

In terms of practical applications, stochastic differential equations can be used to model stock prices, interest rate, and most importantly the Black-Scholes-Merton model. By combining what we learned about stochastic calculus with Calculus IV methods of solving differential equations, we obtained some very useful results. In the case of a stock:

2.1 Model for Stock Price Process

Let us model stock price which is continuous and driven by Brownian n
to assume the drift and volatility of stock is proportional to the price c
may model the stock price as

$$dS_t = \alpha_t S_t dt + \sigma_t S_t dW_t.$$

After applying Ito-Doeblin formula, to the function X_t= Log (S_t), the result is $dX_t = \left(\alpha_t - \frac{\sigma_t^2}{2}\right) dt + \sigma_t dW_t.$

Through some substitution and ODE techniques, the final result is equivalent to

$$S_t = S_0 \exp\left(\int_0^t \left(\alpha_s - \frac{\sigma_s^2}{2}\right) ds + \int_0^t \sigma_s dW_s\right).$$

Aside from stocks, SDEs can also be used to map interest rate models. Specifically, they can analyzed in the case of the Vasicek and Cox-Ingersoll-Ross(CIR) models, which are shown below. A key difference is that the CIR model does not have a closed form solution, and can only really be analyzed for its expectation and variance.

2.2 Vasicek Interest Rate Model

Vasicek interest model is given by SDE

$$dR_t = (\alpha - \beta R_t)dt + \sigma dW_t$$

$$R_t = e^{-\beta t}\left(R_0 + \alpha \int_0^t e^{\beta s} ds + \sigma \int_0^t e^{\beta s} dW_s\right)$$
$$= R_0 e^{-\beta t} + \frac{\alpha}{\beta}(1 - e^{-\beta t}) + \sigma e^{-\beta t} \int_0^t e^{\beta s} dW_s.$$

2.3 Cox-Ingersoll-Ross(CIR) Interest Rate Model

The CIR interest rate model is given by SDE

$$dR_t = (\alpha - \beta R_t)dt + \sigma \sqrt{R_t} dW_t; \quad R_0 > 0.$$

Proposition 2.2. *In CIR model, we have*

$$\mathbb{E}R_t = e^{-\beta t} R_0 + \frac{\alpha}{\beta}(1 - e^{-\beta t})$$
$$Var(R_t) = \frac{\sigma^2}{\beta} R_0 (e^{-\beta t} - e^{-2\beta t}) + \frac{\alpha\sigma^2}{2\beta^2}(1 - 2e^{-\beta t} + e^{-2\beta t})$$

The last important topic from Math Finance I wish to discuss is the well-known known model known as the Black-Sholes-Merton model. First, one must consider the common situation:

3.1 Introduction

We assume that stock price follows geometric Brownian motion

$$dS_t = \alpha S_t dt + \sigma S_t W_t.$$

Our investment strategy is π is the number of share of stock. The remainder of wealth is put in money market with interest rate r. Then, our wealth process X follows

$$dX_t = (X_t - \pi_t S_t) r dt + \pi_t dS_t = (rX_t + \pi_t(\alpha - r)S_t)\, dt + \pi_t \sigma S_t dW_t.$$

If we assume that X_t is a function of time and stock price $c(t,S_t)$, apply Ito-Doeblin formula, and substitute known terms, the final result is:

$$\begin{aligned} dX_t &= c_t(t,S_t)dt + c_x(t,S_t)dS_t + \frac{1}{2}c_{xx}(t,S_t)d[S,S]_t \\ &= \left(c_t(t,S_t) + \frac{1}{2}c_{xx}(t,S_t)\sigma^2 S_t^2\right) dt + c_x(t,S_t)\alpha S_t dt + c_x(t,S_t)\sigma S_t dW_t. \end{aligned}$$

Therefore, $c_x(t,S_t) = \pi_t$ and

$$\begin{aligned} rc(t,S_t) + \pi_t(\alpha - r)S_t &= c_t(t,S_t) + \frac{1}{2}c_{xx}(t,S_t)\sigma^2 S_t^2 + c_x(t,S_t)\alpha S_t \\ &= c_t(t,S_t) + \frac{1}{2}c_{xx}(t,S_t)\sigma^2 S_t^2 + \pi_t \alpha S_t. \end{aligned}$$

Therefore, we are looking for the solution $c \in C^{1,2}$ that satisfies

$$c_t(t,x) + rxc_x(t,x) + \frac{1}{2}\sigma^2 x^2 c_{xx}(t,x) = rc(t,x); \quad t \geq 0, x \geq 0$$

wit terminal condition $c(T,x) = (x-K)^+$. The above PDE is called Black-Scholes-Merton PDE. If we found a solution for this PDE, our portfolio value is $X_t = c(t,S_t)$.

This Partial Differential equation actually has a relatively easy to find solution, which is written below. Using computational tools, one can easily find the d and N values given only a few variables like time, interest rates, and stock volatility.

The solution of Black-Scholes-Merton PDE is

$$c(t,x) = xN(d_+(T-t,x)) - Ke^{-r(T-t)}N(d_-(T-t,x))$$

$$\begin{aligned} d_\pm(\tau,x) &:= \frac{1}{\sigma\sqrt{\tau}}\left[\log\frac{x}{K} + \left(r \pm \frac{\sigma^2}{2}\right)\tau\right] \\ N(y) &:= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{y} e^{-z^2/2}dz \end{aligned}$$

Without a doubt, this course was the most difficult one I have ever taken up to that point in my life. I dedicated the majority of my studying time to it, and still only managed to earn a mid C grade on the first midterm. Thankfully, as the year progressed, I began to see more and more improvement in my ability to learn the material. When the day of my second midterm came, I felt much more prepared, and managed to earn a high B grade. My homework average was high, but that didn't matter as much since it was only 10% of my total grade. Subsequently, I knew that my grade, whether it would be a C or a B, would depend entirely on my final exam.

Truth be told, I have never studied for an exam as thoroughly as I did for Math Finance I. I re-did old exams, old homework problems, re-read and re-wrote the notes, read the textbook in my spare time, etc. Thankfully, my hard work paid off and I managed to score well enough on the final exam that my grade for the year became a B. As a student who is used to receiving As, I have never been so proud of getting a B in my life. It showed that I could handle the most difficult course in the entire Math Finance graduate school, and effectively guaranteed my admission into the program.

Unfortunately, my other grades in my undergraduate classes suffered as a result of my neglecting them in favor of Math Finance I. Though I managed to get my first B in a graduate course, I also received the first C+ of my college career, in Advanced Linear Algebra, and earned the lowest semester GPA of my college career. In spite of these outcomes, I was still happy with my results. I took on a class that challenged me like no other class before it, and managed to come out victorious. More importantly, since I had done well in the course, Professor Triet Pham, who I mentioned above, agreed to write me a recommendation letter for my eventual application to the MSMF program.

IV. Math Finance II

Now that I had completed Math Finance I, I felt confident in my ability to take on the graduate program. In addition to taking Math Finance II, the continuation of the previously discussed course, I also concurrently took Numerical Analysis II and Econometrics I (graduate). To my surprise, the professor teaching the Math Finance II course was none other than Professor Triet Pham. I was very pleased to be able to learn from my favorite teacher, especially since he got me interested in Math Finance in the first place and would be writing my recommendation later in the same semester.

Initially, most of the first quarter of the semester was spent on reviewing Math Finance I. Topics that I mentioned earlier, like measure theory, Brownian motion, Ito calculus, etc. were all discusses again in depth, and I was personally very grateful for the review. However, most of the topics covered in this course are simply applications of what I discussed previously, so I will only mention the main ideas of the course.

The first new concept Professor Pham introduced was the idea of applying the Ito Formula to 'jump processes' like the Poisson process. The details of this can be seen below, and utilize the fact that a stock can be separated into its continuous parts and its jump parts.

1.10.1 Ito's formula for one jump process

The most general jump process we will consider in this chapter has the following form:

$$X(t) = X(0) + \int_0^t \alpha(s)ds + \int_0^t \gamma(s)dWs + J(t),$$

where $J(t)$ is a pure jump process (Discussed in Section (1.6.2)). We also denote by $X^c(t)$ the continuous part of X, that is

$$X^c(t) = X(0) + \int_0^t \alpha(s)ds + \int_0^t \gamma(s)dWs.$$

$$\begin{aligned} f(X(t)) &= f(X(0)) + \int_0^t f'(X(s))dX^c(s) + \int_0^t \frac{1}{2} f''(X(s))\gamma^2(s)ds \\ &\quad + \sum_{0<s\leq t} f(X(s)) - f(X(s-)). \end{aligned}$$

The other unique part of the course was examining the various stock market options that exist throughout the world. To elaborate, options in the American market can be exercised at any time up to the expiration date, European options can only be exercised on the expiration date, and Asian options cannot be exercised at all and are equal to average amount of its worth over a period of time. Each option has a unique partial differential equation that can be used to analyze its dynamics. I have already shown the European model in the form the Black-Sholes Merton, so I will now show the American and Asian option PDEs. Through the Ito-Doeblin formula of each option's unique SDE, and some partial differentiation, the following results are obtained:

Then we have the following PDE for the Asian option, *assuming the condition* $\lim_{y\to\infty} G(x,y) = 0$

$$-rv(t,x,y) + v_t(t,x,y) + v_x(t,x,y)rx + v_y(t,x,y)x + \frac{1}{2}v_{xx}(t,x,y)\sigma^2(t,x)x^2 = 0,$$
$$0 < x, y < \infty, 0 \le t < T;$$
$$v(T,x,y) = G(x,\frac{y}{T});$$
$$v(t,0,y) = e^{-r(T-t)}G(0,\frac{y}{T});$$
$$\lim_{y\to\infty} v(t,x,y) = 0.$$

American Option

u and u' are continuous and u'' is continuous except possibly at a finite number of points, where it has jump discontinuities, and it satisfies

$$ru(x) - rxu'(x) - \frac{1}{2}\sigma^2x^2u''(x) \ge 0; \tag{8.14}$$

on the set where $u(x) > (K - x)^+$ (the continuation set),

$$ru(x) - rxu'(x) - \frac{1}{2}\sigma^2x^2u''(x) = 0. \tag{8.15}$$

Overall, Math Finance II was not nearly as difficult as Math Finance I, and I look forward to taking the final for it soon.

V. Conclusion

As I stated before, my senior year has been the most challenging year of my life. I had to balance several graduate courses, the rest of my undergraduate courses, SAS Honors Program requirements, a part-time job, and Greek life my fraternity AEPI. Thankfully, I was able to emerge from this crucible with positive results. I managed to maintain above a 3.0 GPA in the four graduate courses I completed, and will be graduating in May. However, I was still filled with an overwhelming anxiety because I was not officially accepted into the graduate program. Although I knew it was unlikely, the possibility that I could be declined from the school with 12 credits already completed, which would essentially mean I wasted a year of my life, made me very uncomfortable. Fortunately, on April 1st, I received the following email:

> Dear Mikhail Fishbeyn,
>
> Congratulations! I am delighted to inform you that our Admissions Committee has recommended to our Graduate Admissions office that you be admitted to our Master of Science Degree in Mathematics Program with Option in Mathematical Finance (MSMF) at Rutgers University-New Brunswick, for the program beginning this Fall.

I have never been so relieved in my life than when I saw this email. Apart from quelling my nervous thoughts, it also validated the last year of my life, and gave me a clear picture of what my next few years will look like.

Thanks to the alternative Capstone option, I was able to complete both my SAS Honors requirements and get a significant head start on my graduate education. Also, since I took the previously mentioned courses as an undergraduate, I was able to save an absurd amount of money due to my Trustee Scholarship. I learned very interesting and useful applications of advanced mathematics and am still eager to learn more next semester. I only have another 18 credits left to complete, which I intend to finish within a year of my graduation. I am very happy that I ended up at Rutgers, and even more so that I was able to take part in both the SAS Honors program and the Accelerated MSMF program. I was one of the only, if not the only, students that were able to participate in both these prestigious programs, and I am very grateful for the experience. I now feel ready to take on the challenges of post-education, since nothing can be harder than the challenges I endured this year.

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